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HEURISTICS FOR ONE CLASS OF MINIMAL COVERING PROBLEM IN CASE OF LOCATING UNDESIRABLE GOODS

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Abstract
Modern society has experienced an increasing public awareness of environmental issues. In particular, problems within the field of transportation and the location of dangerous goods facilities have attracted considerable public attention. In this paper, the specific problem of locating undesirable goods has been considered. Such facilities should be located in some of the network nodes belonging to a known discrete set, under conditions of minimal safety distance, both between the warehouse facilities themselves and between the warehouses and other neighboring objects. The objective here is to maximize the quantity of goods stored while at the same time respecting minimal safety distances. This paper presents the problem formulation and proposes heuristic solution approaches which are tested on numerous examples.

Keywords*: undesirable good, location, safety distance, heuristic*

1. INTRODUCTION

Many facilities which are widely used and provide a service are known as "undesirable" or "obnoxious". Examples of undesirable facilities include solid waste repositories, polluting plants, radioactive waste storage sites and explosive storage sites, as well as noise, odor or heat emitters. Those facilities generate different undesirable effects which can be felt over a certain geographical space, and making decisions about their spatial position is crucial when it comes to minimizing the environmental risks. This is why one of the most active research areas within location theory in recent years has been the location of undesirable or obnoxious facilities.

Numerous location models in this area use a single objective function that either maximizes the sum of the distances between the facilities and the demand points (maxisum criterion), or maximizes the minimal distances between the undesirable facilities and the customers (maximin criterion). A very comprehensive review of distance maximization models for undesirable single facilities, as well as multifacility locations can be found in [5]. A variety of single and multi criteria methods have been developed depending on the problem statement [1], [2], [3], [5], [6] and [7]. Another area of research is the minimization of the total population affected by the facility which leads to a minimal covering problem. Finding the circle of given radius R covering the least total (minisum) or maximum (minimax) weight of the destinations is studied in [4]. In [8] it is shown how both of the aforementioned problems may be solved simultaneously by the bicriteria model.

However, in spite of fact that literature offers numerous different approaches and models devoted to the location of undesirable facilities, many problems in this area remain undiscovered, and have not yet been analyzed.

This paper, which is an extension of previous researches [9], [10], considers problem of locating facilities for storing dangerous goods within a "smaller" closed region, destined only for this purpose. More precisely, the area designated for storing undesirable facilities is inside a region which only has nodes which are candidates for locating storage objects. Outside[1] the region are neighboring facility nodes which despite not being destined for storing materials, must also be made safe from the undesirable effects of the stored materials.

The objective is to maximize the quantity of material stored while at the same time respecting minimal safety distances, both between the storage facilities themselves, and between the storage facilities and any neighboring objects.

The region itself, because of its dimensions which may be neglected when compared with the service region, could be assumed to be a "service point" for all of the assigned customers, and from there customer nodes, as well as transportation costs may be excluded from any further analysis.

The rest of the paper is organized as follows. Chapter 2 introduces the problem in more detail and presents the formulation of the location problem. Chapter 3 proposes heuristic algorithms and presents numerical example of their application, Chapter 4 presents heuristic algorithms performances and Chapter 5 offers some concluding remarks.

2. PROBLEM DESCRIPTION AND FORMULATION

Storing and keeping dangerous goods like explosives, flammable materials and compressed gasses is characterized by the opportunity to transfer any undesirable effects to the objects in the neighborhood thus causing destruction, serious damage and fire in these areas. Those effects spread spherically from the source, reaching the surrounding object within a certain radius known as the minimal safety distance. The minimal safety distance usually depends on the quantity and characteristics of the activated material, as well as other relevant characteristics (building construction, the mutual spatial position of the donor and acceptor objects...). In this paper, the relations between the safety distance,

[1] Those nodes could be situated inside the region too, but because of simplicity it is assumed here that they are only outside the region.

the material characteristics and the quantity used correspond to explosives, although similar relations may be assumed in cases of other obnoxious facilities. Based on this assumption, the minimal safety distance may be determined as [9,10]:

- constant value, which depends only on the explosive's characteristics;
- the function of the net explosive weight for quantity (Q). In this case the minimal safety distance R may be calculated by using $R=P\cdot Q^{1/3}$. Here, R is the minimal safety distance required; P is the protection factor depending on the degree of risk assumed or permitted, independent of the kind of explosive, and Q is the net explosive weight for the given quantity of a certain explosive.

In this paper the location problem when the safety distance does not depend on the quantity stored, and which has a constant value for a certain class of materials have been analyzed.

Let G=(N,A) be a network, where $N=\{1, ..., i, j, ..., n\}$ is the set of nodes, and A is the set of arcs (i,j). To each arc $(i,j)\in A$ a nonnegative scalar c_{ij} is associated which represents the euclidean distance between nodes i,j. Set N is partitioned into two subsets $N=N_I\cup N_E$, where N_I represents the nodes inside the region which are the candidates for storing materials, and N_E represents the outside nodes which are not designated for storing materials, but must be kept at a safe distance from the materials stored in the inside nodes. Two scalars R^I, R^E are associated to material stored which represents the minimal safety distances for internal facilities, and neighboring objects respectively. To each node $i\in N_I$ is the associated weight v_i, which represents the node's capacity restriction for material stored.

Based on previous notations, QID (**Q**uantity **I**ndependent **D**istance) location problem, in case when only one undesirable material should be stored, analyzed here may be formulated as the following nonlinear 0-1 programming problem:

$$\text{Maximize } ZQI=\sum_{i\in N_I} v_i \cdot x_i \tag{2.1}$$

subject to:

$$x_i \cdot x_j \cdot (c_{ij} - R_k^I) \geq 0,\ \forall\, x_i, x_j;\ i, j \in N_I \tag{2.2}$$

$$c_{ij} - R^E \geq 0,\ \forall\, i \in N_I,\ j \in N_E \tag{2.3}$$

$$x_i \in \{0,1\},\ i \in N_I \tag{2.4}$$

The objective function, presented by Eq. (2.1), is to maximize the quantity of undesirable material stored. The Eq. (2.2) provides that each pair of nodes where undesirable materials are stored is mutually at the correct safety distance. The Eq. (2.3) provides that the all candidate nodes for storing undesirable material are at the correct safety distance from the facilities outside the region. The Eq. (2.4) defines the values of the decision variable.

To solve the problem, QID heuristic has been proposed.

3. SOLUTION APPROACH

Let $\Omega^* \subseteq N_I$ be the set of nodes included in the solution, $\Omega_t^* \subseteq \Omega^*$ be the set of nodes included in the solution in t-th iteration, $E\subseteq N_I$ be the set of nodes excluded from the solution, $E_t \subseteq N_I$ be the set of nodes which are excluded from the solution in t-th iteration, and let $A_t \subseteq N_I$ be the set of "active nodes" in t-th iteration.

For the arbitrary node $i \in A_t$, let $\Pi(i)$ be the subset of all nodes $j \in A_t$, which are at the distance from node i, less than the minimal safety distance ($c_{ij} < R^I$). That is:

$$\Pi(i) = \left\{ j \mid c_{ij} < R^I,\ j \in A_t \right\} \tag{3.1}$$

For the arbitrary node $i \in A_t$ the following measurements have also been defined:

$$\alpha(i) = v_i - \sum_{j \in \{\Pi(i) \setminus i\}} v_j \tag{3.2}$$

$$\beta(i) = v_i + \sum_{j \in \{A_t \setminus \Pi(i)\}} v_j \tag{3.3}$$

$$w\alpha(i) = v_i - \frac{v_i}{\sum_{i \in A_t} v_i} \sum_{j \in \{\Pi(i) \setminus i\}} v_j \tag{3.4}$$

$$w\beta(i) = v_i + \frac{v_i}{\sum_{i \in A_t} v_i} \sum_{j \in \{A_t \setminus \Pi(i)\}} v_j \tag{3.5}$$

$$\alpha\beta(i) = v_i + \sum_{j \in \{A_t \setminus \Pi(i)\}} v_j - \sum_{j \in \{\Pi(i) \setminus i\}} v_j \tag{3.6}$$

$$w\alpha\beta(i) = v_i + \frac{v_i}{\sum_{i \in A_t} v_i} \left(\sum_{j \in \{A_t \setminus \Pi(i)\}} v_j - \sum_{j \in \{\Pi(i) \setminus i\}} v_j \right) \tag{3.7}$$

Obviously, α**(i)** summarizes the weight of node $i \in N_I$ and the negative weights of all those nodes which at the distance from node $i \in N_I$ are less than the minimal safety distance. Therefore, $\alpha(i)$ represents "utility" gained by storing maximum quantity of dangerous goods in the node $i \in N_I$, which is calculated as a result of subtraction of endangered nodes capacities from capacity of the node $i \in N_I$. Measurement β**(i)** summarizes the weight of node $i \in N_I$ and the weights of all nodes which at the distance from node $i \in N_I$ are either equal to or greater than the minimal safety distance. $\beta(i)$ represents "utility" gained by storing maximum quantity of dangerous goods in the node $i \in N_I$, which is calculated as the sum of capacity of the node $i \in N_I$ and the capacity of nodes which are not affected by storing dangerous material in the node $i \in N_I$. Measurement **w**α**(i)** is modified (weighted) measurement $\alpha(i)$ in which the sum of negative weights of all those nodes which at the distance from node $i \in N_I$ are less than the minimal safety distance is multiplied by relative weight of the node $i \in N_I$. Measurement **w**β**(i)** is modified (weighted) measurement $\beta(i)$ in which the sum of weights of all nodes which at the distance from node $i \in N_I$ are either equal to, or greater than the minimal safety distance is multiplied by relative weight of the node $i \in N_I$. The weight of the node $i \in N_I$ has been emphasized within both measurements. Measurement $\alpha\beta$**(i)** represents total utility calculated by adding the weight of the node $i \in N_I$, the weights of all nodes which at the distance from the node $i \in N_I$ are either equal to or greater than the minimal safety distance and the negative weights of all those nodes which at the distance from node $i \in \mathbf{N}_I$

are less than the minimal safety distance. Measurement **w$\alpha\beta$(i)** is modified (weighted) measurement $\alpha\beta$(i) according to the same principle as wα(i) and wβ(i).

Based on those measurements, six variations of QID heuristic algorithm are proposed: QIDα, QIDβ, wQIDα, wQIDβ, QID$\alpha\beta$ and wQID$\alpha\beta$. Those algorithms differ from each other only in the measurements applied, therefore they are presented as unified following algorithm.

3.1. QID HEURISTIC ALGORITHMS

STEP 1 Set $t = 1$

STEP 2 Set $\Omega_t^* = \varnothing$, $E_t=\varnothing$, $A_t=N_I$

STEP 3 For each node $i \in A_t$, define set $\Pi(i)$ according to Eq. (3.1)

STEP 4 For each node $i \in A_t$, calculate utility: $\alpha(i)$ according to Eq. (3.2) for QIDα heuristic; $\beta(i)$ according to Eq. (3.3) for QIDβ heuristic; wα(i) according to Eq. (3.4) for wQIDα heuristic; wβ(i) according to Eq. (3.5) for wQIDβ heuristic; $\alpha\beta$(i) according to Eq. (3.6) for QID$\alpha\beta$ heuristic; w$\alpha\beta$(i) according to Eq. (3.7) for wQID$\alpha\beta$ heuristic

STEP 5 Find node i^* with the maximal utility value of: $\alpha(i^*) = \max_{i \in A_t} [\alpha(i)]$ for QIDα heuristic; $\beta(i^*) = \max_{i \in A_t} [\beta(i)]$ for QIDβ heuristic; $w\alpha(i^*) = \max_{i \in A_t} [w\alpha(i)]$ for wQIDα heuristic; $w\beta(i^*) = \max_{i \in A_t} [w\beta(i)]$ for wQIDβ heuristic; $\alpha\beta(i^*) = \max_{i \in A_t} [\alpha\beta(i)]$ for QID$\alpha\beta$ heuristic; $w\alpha\beta(i^*) = \max_{i \in A_t} [w\alpha\beta(i)]$ for wQID$\alpha\beta$ heuristic. If two or more nodes have the same maximal utility value of: $\alpha(i)$; $\beta(i)$; wα(i); wβ(i); $\alpha\beta$(i); w$\alpha\beta$(i), node i^* should be chosen arbitrarily.

STEP 6 Set $t = t + 1$

STEP 7 Update set Ω_t^*: $\Omega_t^* = \Omega_{t-1}^* \cup i^*$

STEP 8 Update set E_t: $E_t=E_{t-1}\cup\{j \mid j\in\Pi(i^*)\backslash i^*\}$

STEP 9 Update set A_t: $A_t=A_{t-1}\backslash\{E_t \cup i^*\}$

STEP 10 If $A_t\neq\varnothing$, proceed with **STEP 3**, otherwise go to **STEP 11**

STEP 11 Set $\Omega^* = \Omega_t^*$, $E=E_t$

STEP 12 Calculate the objective function value ZQI= $\sum_{i\in\Omega^*} v_i$

STEP 13 End algorithm.

3.2. QID HEURISTIC APPLICATION EXAMPLE

The proposed heuristic is demonstrated in the following example. Set N_I has been given by nodes' coordinates N_I={(3,20), (61,1), (48,42), (16,54), (89,77), (81,65), (138,59) and (150,19)}.

Node weights are $||v_j||=||79, 47, 56, 55, 67, 33, 56$ and $12||$. Let R^I=61. The illustrative problem, the iterations of the QIDα heuristic algorithm application and the results are shown in **Fig 1**.

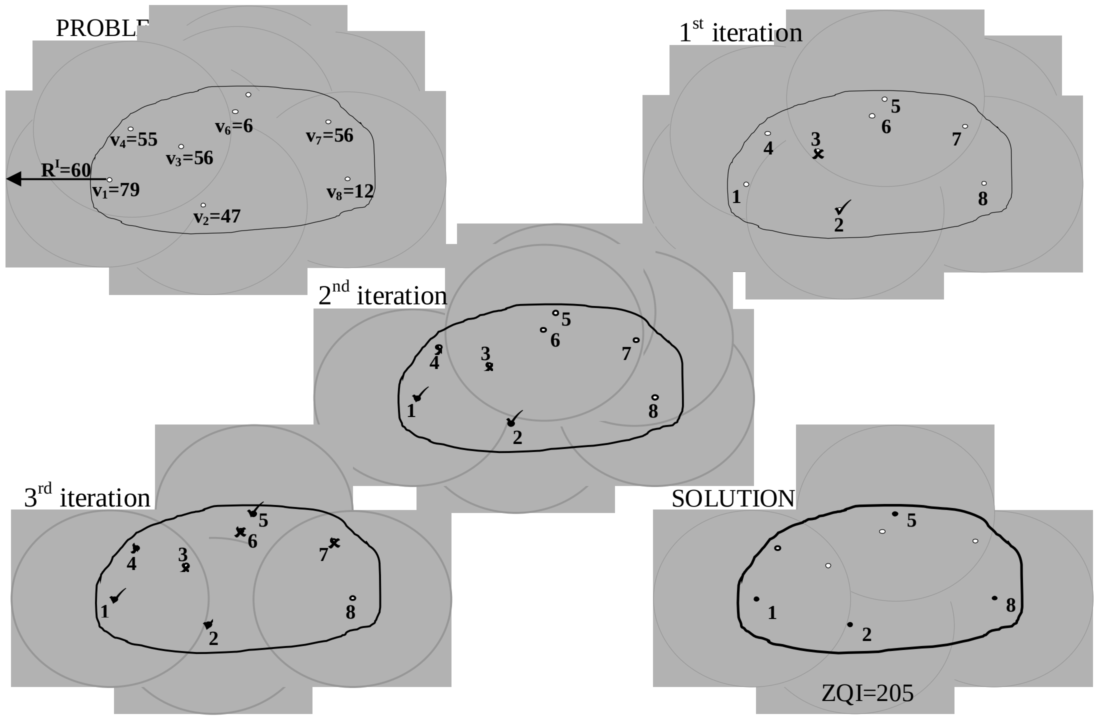
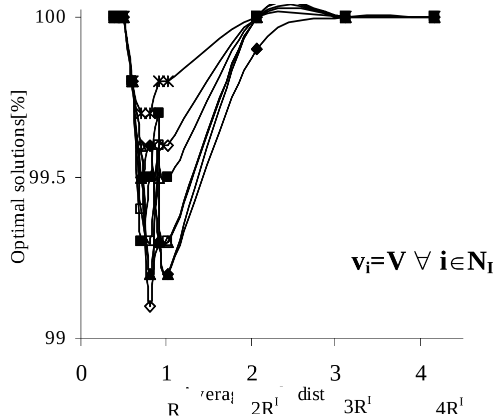


Figure 1 The QIDα application example

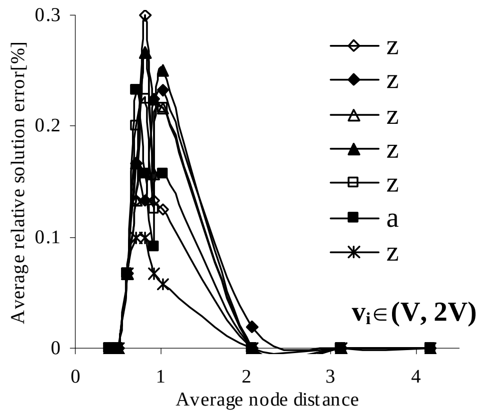
4. QID HEURISTIC PERFORMANCES

The proposed heuristic algorithms were also tested on three groups of 30000 randomly generated problems. The first group comprised of problems with 8 candidate nodes, the second with 12 and the third with 16 candidate nodes. In all generated problems, the value of minimal safety distance is the same, R^I=const. Each group of 30000 problems is divided into a three 10000 problems subgroups: within the first subgroup the weights of all nodes are equal, i.e. $v_i=V$, $i\in N_i$; within the second subgroup the weights of nodes are generated in the range from V to 2V; within the third subgroup the weights of nodes are generated in the range from V to 10V. Further, each subgroup is divided into a 10×1000 problems in which the average node distance is varied in range from $0.4R^I$ to $4.2R^I$.

Percentage of the optimal solutions and percentage of the average relative solution error based on 90000 numerical examples are shown in Fig.4.1, Fig. 4.2. and Fig. 4.3., for the group problems with 8, 12 and 16 nodes, respectively. Those performances are also shown for “integrated” QID (IQID) heuristic, by the same figures, as a result of finding the best of all generated problems solutions calculated for all heuristics.

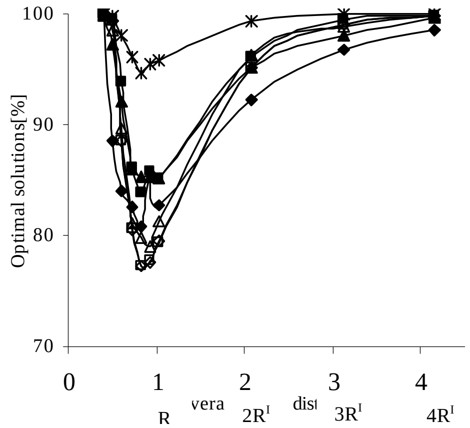


100
99.5
99
0
1
2
3
4
Optimal solutions[%]
vi=V ∀ i∈NI
R
2R
3R
4R
QIDβ
wQIDβ
QIDα
wQIDα
QIDαβ
wQIDαβ
IQID
R
2R
3R
4R

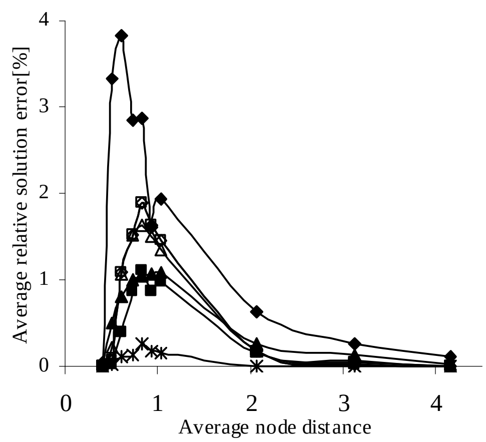


0.3
0.2
0.1
0
0
1
2
3
4
Average relative solution error[%]
Average node distance
z
z
z
z
z
a
z
vi∈(V, 2V)

$v_i \in (V, 10V)$


Optimal solutions[%]
100
90
80
70
0
1
2
3
4
R
vera
2R^I
dist
3R^I
4R^I

R 2R^I 3R^I 4R^I


Average relative solution error[%]
4
3
2
1
0
0
1
2
3
4
Average node distance

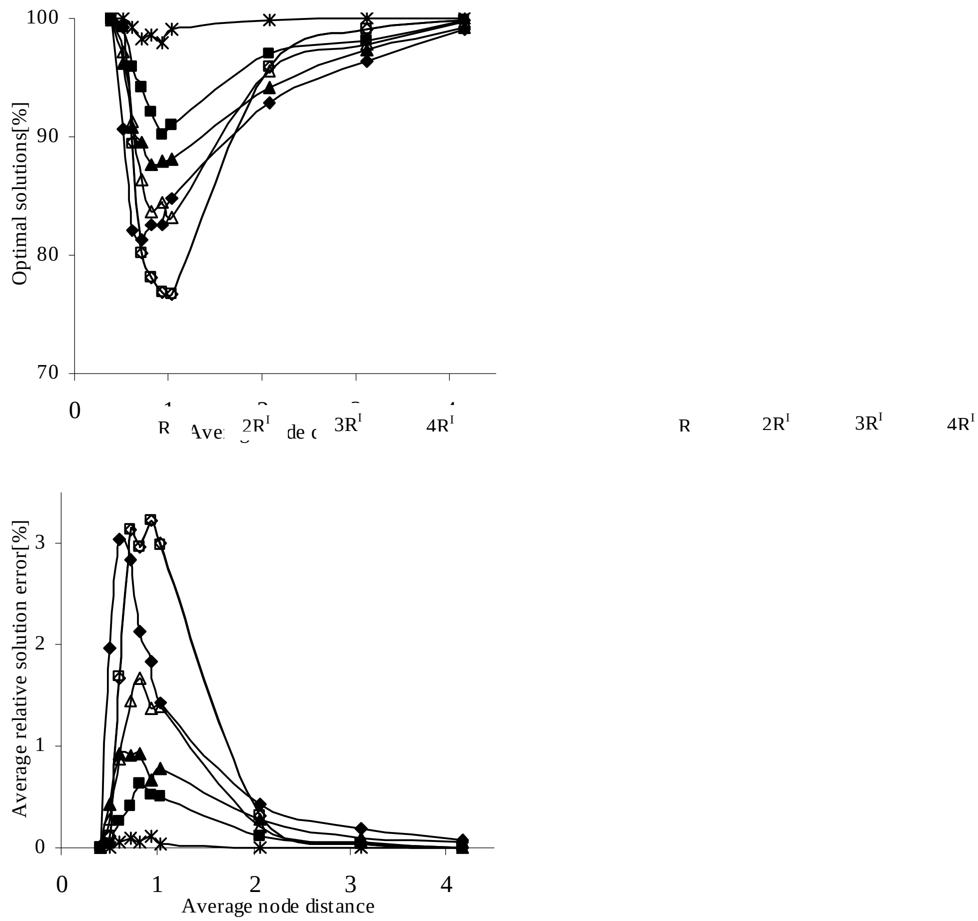
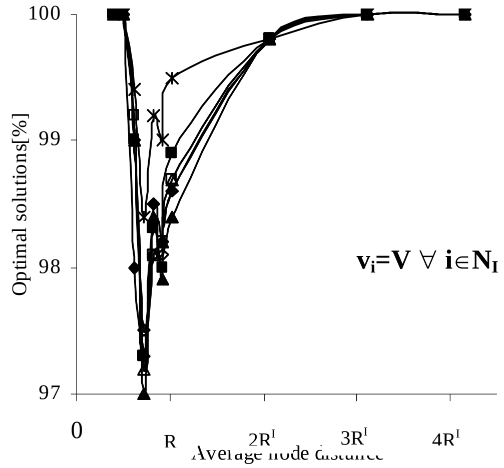


Fig. 4.1. QID heuristic algorithms performances for problems with 8 candidate nodes



QIDβ

wQIDβ

QIDα

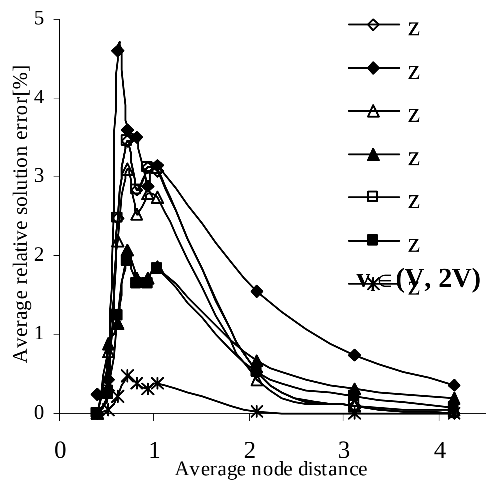
wQIDα

QIDαβ

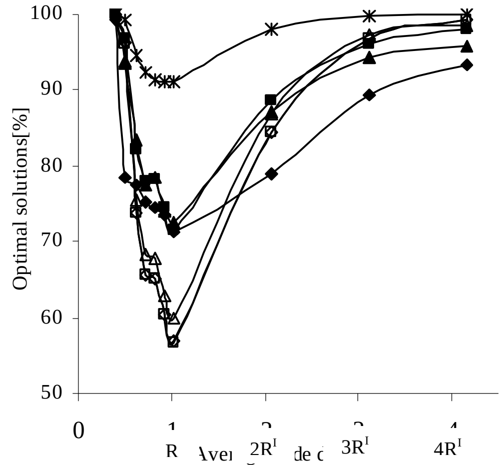
wQIDαβ

IQID

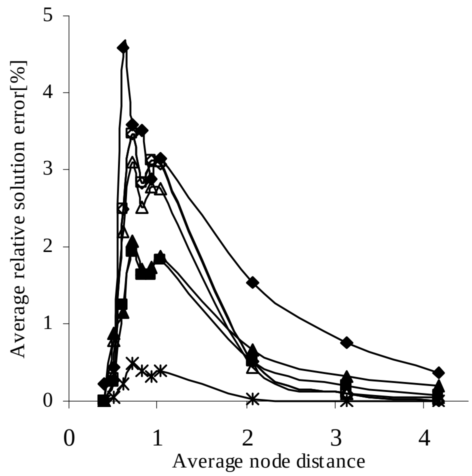
R 2R[I] 3R[I] 4R[I]

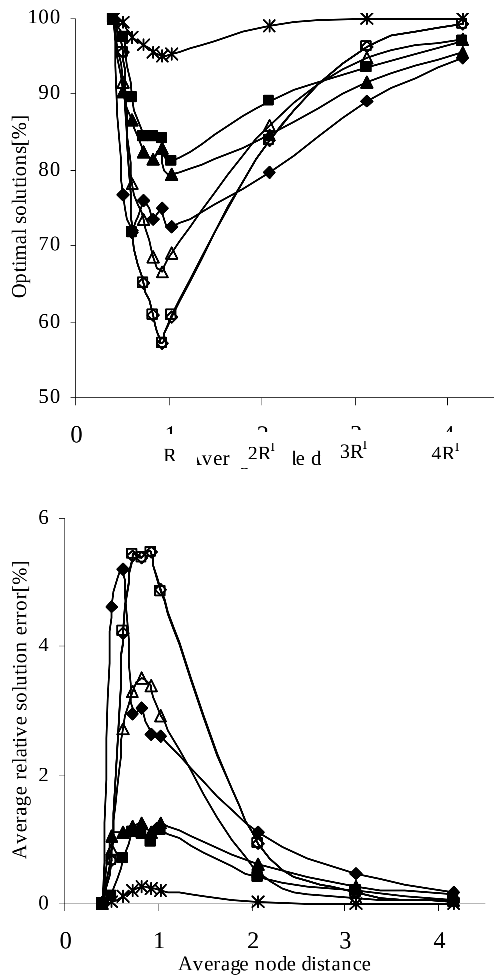


v_i∈(V, 10V)


Optimal solutions[%]
100
90
80
70
60
50
0
R
2RI
3RI
4RI

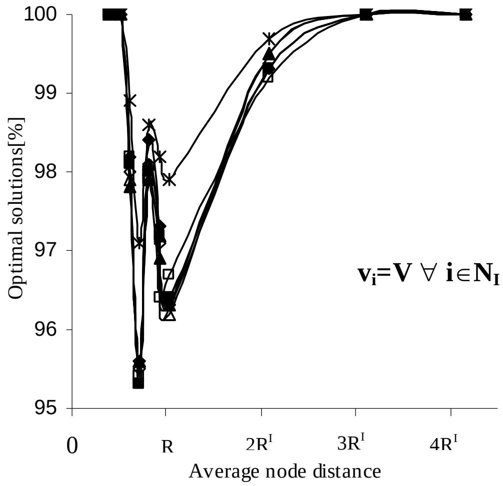
R 2RI 3RI 4RI


Average relative solution error[%]
5
4
3
2
1
0
0
1
2
3
4
Average node distance



R 2R' 3R' 4R'

Figure 4.2 QID heuristic algorithms performances for problems with 12 candidate nodes



QIDβ

wQIDβ

QIDα

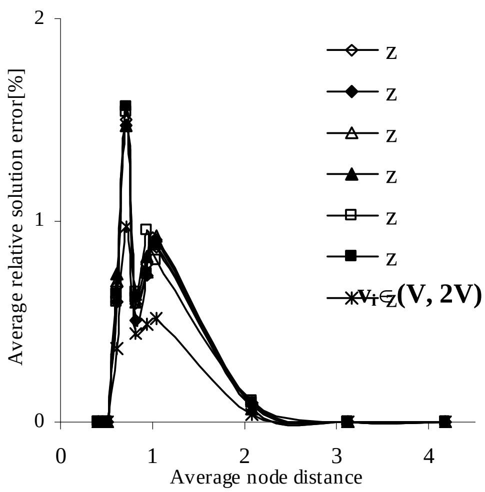
wQIDα

QID$\alpha\beta$

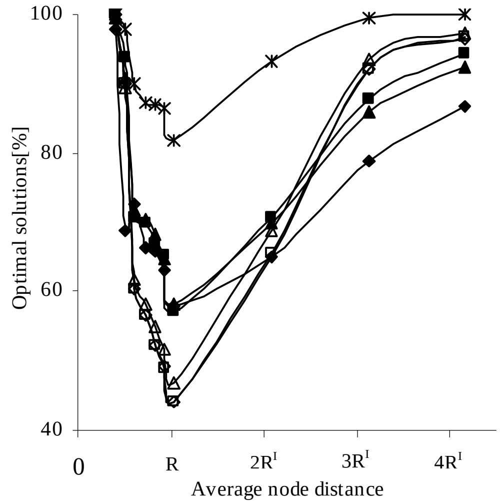
wQID$\alpha\beta$

IQID

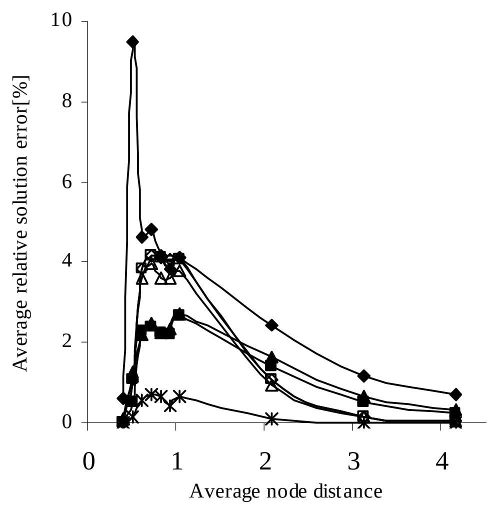
R 2R[I] 3R[I] 4R[I]

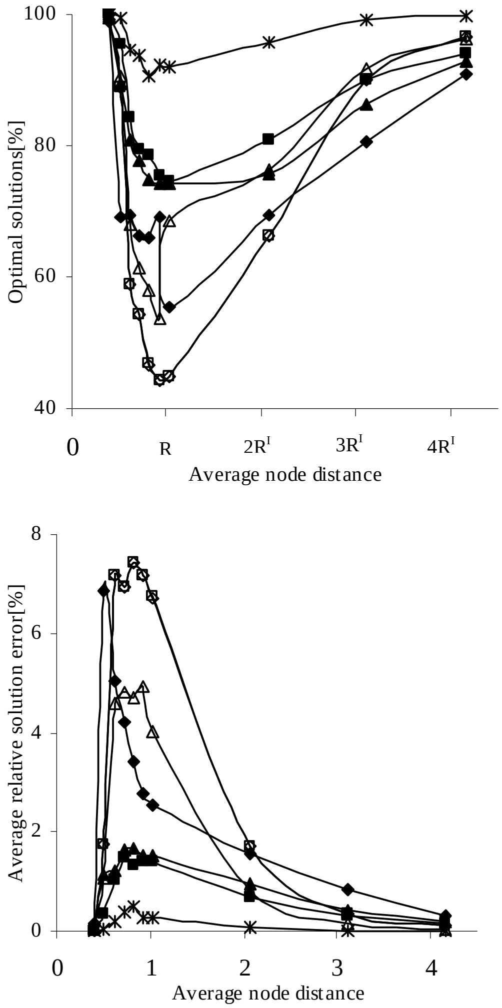


$v_i \in (V, 10V)$


100
80
60
40
Optimal solutions[%]
0
R
$2R^{I}$
$3R^{I}$
$4R^{I}$
Average node distance

R $2R^{I}$ $3R^{I}$ $4R^{I}$


10
8
6
4
2
0
Average relative solution error[%]
0
1
2
3
4
Average node distance



R 2R^I 3R^I 4R^I

Figure 4.3 QID heuristic algorithms performances for problems with 16 candidate nodes

Generally, from Figure 4.1 to Figure 4.2 it is noticeable that the worst heuristics performances are in the cases of average node distance are almost the same with the value of R^I. Percentage of optimal solutions, in the cases of equal nodes weight, is rather unified for all the heuristics and has not dropped below 95%, and in the cases of IQID has not dropped below 97%. Further to this problems subgroup, maximal percentage of relative solution errors of heuristic algorithms applications is 1.5%, as for the IQID is less then 1%. In other two problem subgroups, v_i=[V-2V], v_i=[V-10V], these performances differs from previous subgroup. For example, for problems with 16 nodes, minimal percentage of optimal solutions calculated by IQID heuristic is 86.3%, for v_i=[V-2V], while for v_i=[V-10V] is 90.7%. Maximal percentage of relative solution errors calculated by IQID heuristic for the same problem group is 0.64% for v_i=[V-2V], as for v_i=[V-10V] is 0.49%. Relatively small differences in between results performances within these problem subgroups show substantial stability of IQID heuristic in cases of nodes weight changes. These differences are consequence of the increased results calculated by weighted heuristics (wQIDα, wQIDβ, wQIDαβ), when nodes weight range is wider. To compare, for subgroup v_i=[V-10V], of problem group with 16 nodes, minimal percentage of optimal solutions calculated by weighted heuristics is: 55.5%, 74.3% and 74.6%, in case of v_i=[V-10V], for wQIDα, wQIDβ and wQIDαβ respectively, while in case of v_i=[V-2V] this performance takes values as follows: 58%, 58.2% and 57%. Heuristics QIDβ and QIDαβ, in all generated examples produce results with neglected differences. It is interesting that results calculated by these two heuristics, for subgroups v_i=[V-2V] and v_i=[V-10V], show stability toward nodes weight changes. At the same time average results calculated by these heuristics, comparing with other heuristics, are of the least precision.

For testing the heuristics a special program has been created in programming language DELPHI V5.0 (Build 5.6.2.). On PC Pentium 4, 1.5 GHz with 512 MB SDRAM and BUS speed 100 MHz, solution calculation per one heuristic for problem with 500 nodes takes 8 sec and 156 msec, which includes recording calculation results into the outgoing file, in this case 25.1 MB. To compare, solution calculation per one heuristic for problem with 16 nodes takes 16 msec, resulting with outgoing file of 21.8 KB, which indicates that calculation per one node takes just a bit less then a 1 msec, while the rest of the time is spent on recording results in outgoing file.

5. CONCLUDING REMARKS

In this paper, formulation for the problem of locating undesirable facilities when the safety distance does not depend on quantity stored has been considered. To solve the problem six efficient and easy to use heuristic algorithms have been proposed and their performances were subsequently tested on numerous examples. The results are promising and encourage further research.

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BIBLIOGRAPHY

[1] Brimberg J, Wesolowsky G (1995), The rectilinear distance minisum problem with minimum distance constraints, Location Science 3, 203-215;

[2] Drezner Z, Wesolowsky G (1980), A maximin location problems with maximum distance constrains, AIIE Transaction 12, 249-252;

[3] Drezner Z, Wesolowsky G (1985), Location of multiple obnoxious facilities, Transportation science 19, 193-202;

[4] Drezner Z, Wesolowsky G (1994), Finding the circle or rectangle containing the minimum weight of points, Location Science 2, 83-90;

[5] Erkut E, Neuman S (1989), Analytical models for locating undesirable facilities, European Journal of Operational Research 40, 275-291;
[6] Konforty Y, Tamir A (1997), The single facility location problem with minimum distance constraints, Location Science 5, 147-163;
[7] Munoz-Perez J, Saameno-Rodriguez JJ (1999), Location of an undesirable facility in a polygonal region with forbidden zones, European Journal of Operational Research 114, 372-379;
[8] Plastria F, Carrizosa E (1999), Undesirable facility location with minimal covering objectives, European Journal of Operational Research 119, 158-180;
[9] Vidović M, Cvetic S (1996), Locating hazardous materials when the safety distance depends on the quantity stored, Armed Forces professional scientific journal Vol XLIV No.1, 23-34;
[10] Vidović M, Dimitrijevic B (2004), Locating hazardous materials when safety distance depends on quantity stored, Conference book from the Annual International Conference of the German Operations Research Society (GOR) Jointly organized with the Netherlands Society for Operations Research (NGB), Tilburg, Netherlands.